TYPE 1 : (a) PARTIAL FRACTION (5)
(b) Integration. (5)

**2** Let $f(x) = \dfrac{7x+4}{(2x+1)(x+1)^2}$. $\equiv \dfrac{2}{2x+1} - \dfrac{1}{x+1} + \dfrac{3}{(x+1)^2}$

**(i)** Express f(x) in partial fractions. [5]

**(ii)** Hence show that $\int_0^2 f(x)\,dx = 2 + \ln\frac{5}{3}$. [5]

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Integration.

$$\int\left(\frac{2}{2x+1} - \frac{1}{x+1} + \frac{3}{(x+1)^2}\right)dx$$

$$\int\frac{2}{\boxed{2x+1}}\,dx - \int\frac{1}{\boxed{x+1}}\,dx + \int\frac{3}{(x+1)^2}$$

$$\ln(2x+1) - \ln(x+1) + 3\int\frac{1}{(x+1)^2}\,dx$$

Caution: use power operator.

$$\ln\left(\frac{2x+1}{x+1}\right) + 3\int 1(\boxed{x+1})^{-2}\,dx$$

$\square = x+1$

$\square' = 1$

Simplification.

$$\ln\left(\frac{2x+1}{x+1}\right) + \frac{3(x+1)^{-1}}{-1}$$

$$\left|\ln\left(\frac{2x+1}{x+1}\right) - \frac{3}{(x+1)}\right|_0^2$$

$$\left|\left(\ln\left(\frac{2(2)+1}{2+1}\right)-\frac{3}{2+1}\right)-\left(\ln\left(\frac{2(0)+1}{0+1}\right)-\frac{3}{0+1}\right)\right|$$

$$\ln\left(\frac{5}{3}\right)-1-\underset{\text{zero}}{\ln(1)}+3$$

$$\ln\left(\frac{5}{3}\right)+2$$

$$2+\ln\left(\frac{5}{3}\right)$$

**6** Show that $\displaystyle\int_0^7 \frac{2x+7}{(2x+1)(x+2)}\,dx = \ln 50.$ [7]

(W1)(partial)

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(W1) $$\frac{2x+7}{(2x+1)(x+2)} \equiv \frac{4}{2x+1}-\frac{1}{x+2}$$

$$\int\left(\frac{4}{2x+1}-\frac{1}{x+2}\right)dx$$

$$\frac{4}{2}\int\frac{2\times 1}{\boxed{2x+1}}\,dx-\int\frac{1}{\boxed{x+2}}\,dx$$

$$\Big|\,2\ln(2x+1)-\ln(x+2)\,\Big|_0^7$$

$$\Big|\left(2\ln(2(7)+1)-\ln(7+2)\right)-\left(2\ln(2(0)+1)-\ln(0+2)\right)\Big|$$

$2\ln(15) - \ln(9) - 2\ln 1 + \ln 2$

$\ln 225 - \ln 9 + \ln 2$

$\ln\left(\frac{225 \times 2}{9}\right)$

$\ln 50$

**3** Let $f(x) \equiv \dfrac{x^2 + 3x + 3}{(x+1)(x+3)}$. $\rightarrow$ improper $\rightarrow$ long division $\rightarrow$ partial fraction

**(i)** Express $f(x)$ in partial fractions. [5]

**(ii)** Hence show that $\int_0^3 f(x)\,dx = 3 - \frac{1}{2}\ln 2$. [4]

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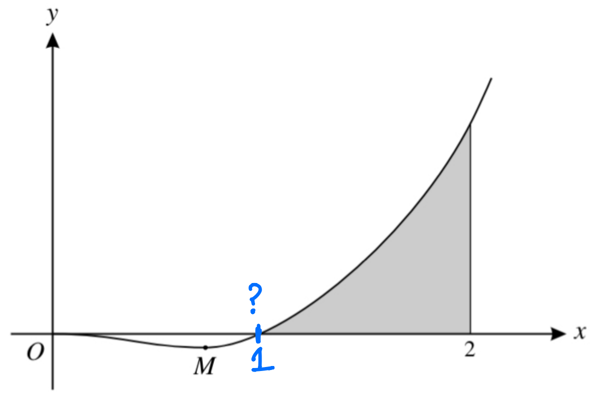
$\boxed{\text{TYPE 2}}$

(a) Differtiation (Stationary Point) (Diff Type 1)

(b) Integration (Area under graph / volume of rotation).

**7**




$y = 0$

$0 = x^3 \ln x$

$0 = \ln x$

e

$x = e^0$

$x = 1$

The diagram shows the curve $y = x^3 \ln x$ and its minimum point $M$.

**(i)** Find the exact coordinates of $M$. [5]

**(ii)** Find the exact area of the shaded region bounded by the curve, the $x$-axis and the line $x = 2$. [5]

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(i) $y = x^3 \ln x$

$$\frac{dy}{dx} = x^3\left[\frac{1}{x} \times 1\right] + \ln x\left[3x^2\right]$$

$$\frac{dy}{dx} = x^2 + 3x^2 \ln x$$

$$0 = x^2 + 3x^2 \ln x$$

$$0 = x^2(1 + 3\ln x)$$

From diagram we know that $x$ cannot be zero. Hence you are allowed to divide $x^2$ on other side.

| RULE |
|---|
| You cannot divide with a variable term unless you are sure that it is non-zero |
| You are not allowed to divide with zero. |

$$0 = 1 + 3\ln x$$

$$3\ln x = -1$$

$$\ln x = \frac{-1}{3}$$

$$\boxed{x = e^{-\frac{1}{3}}}$$

$$y = x^3 \ln x$$

$$y = \left(e^{-\frac{1}{3}}\right)^3 \ln e^{-\frac{1}{3}}$$

$$= e^{-1} \times \frac{-1}{3} \ln e$$

$$= \frac{1}{e} \times \frac{-1}{3}$$

$$= \frac{-1}{3e}$$

$$M = \left( e^{-\frac{1}{3}}, -\frac{1}{3e} \right)$$

(ii) $$\text{Area} = \int_1^2 \underbrace{x^3}_{v} \underbrace{\ln x}_{u} \, dx$$

Diff OF U

$$\ln x \longrightarrow \frac{1}{x} \times 1 = \boxed{\frac{1}{x}}$$

INTEG OF V

$$\int x^3 dx = \boxed{\frac{x^4}{4}}$$

$$u \int v dx - \int \left[ \frac{du}{dx} \times \int v dx \right] dx$$

$$(\ln x)\left(\frac{x^4}{4}\right) - \int \left[ \frac{1}{x} \times \frac{x^4}{4} \right] dx$$

$$\frac{x^4 \ln x}{4} - \frac{1}{4}\int x^3 dx$$

$$\frac{x^4 \ln x}{4} - \frac{1}{4}\left(\frac{x^4}{4}\right)$$

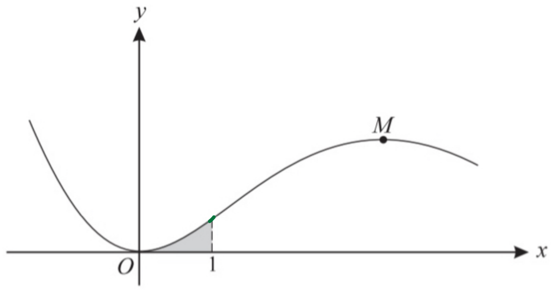
$$\left|\frac{x^4}{4}\left(\ln x - \frac{1}{4}\right)\right|_1^2$$

$$\left|\frac{2^4}{4}\left(\ln 2 - \frac{1}{4}\right) - \frac{1^4}{4}\left(\ln 1 - \frac{1}{4}\right)\right|$$

$$4\left(\ln 2 - \frac{1}{4}\right) - \frac{1}{4}\left(0 - \frac{1}{4}\right)$$

$$4\ln 2 - 1 + \frac{1}{16}$$

$$\boxed{4\ln 2 - \frac{15}{16}}$$




The diagram shows the curve $y = x^2 e^{-\frac{1}{2}x}$.

**(i)** Find the $x$-coordinate of $M$, the maximum point of the curve. [4]

**(ii)** Find the area of the shaded region enclosed by the curve, the $x$-axis and the line $x = 1$, giving your answer in terms of e. [5]

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(i) $y = x^2 e^{-\frac{1}{2}x}$

$$\frac{dy}{dx} = x^2\left[e^{-\frac{1}{2}x} \times -\frac{1}{2}\right] + e^{-\frac{1}{2}x}\left[2x\right]$$

$$\frac{dy}{dx} = e^{-\frac{1}{2}x}\left(-\frac{x^2}{2} + 2x\right)$$

$$0 = e^{-\frac{1}{2}x}\left(-\frac{x^2}{2} + 2x\right)$$

$\boxed{e^{\square} \text{ is always non zero}}$

$$0 = -\frac{x^2}{2} + 2x$$

$$\frac{x^2}{2} = 2x$$

$$x^{\cancel{2}} = 4\cancel{x}$$

Since x is non-zero you can cancel with x.

$$\boxed{x = 4}$$

(ii) $\text{Area} = \int_0^1 \underbrace{x^2}_{u} \underbrace{e^{-\frac{1}{2}x}}_{v} \, dx$

$\int_0^1 \boxed{x}^{2} e^{-\frac{1}{2}x} \, dx$

$\square = x$

$\square' = 1$

reject.

| DIFF OF U | INTEG OF V |
|---|---|
| $x^2 \longrightarrow 2x$ | $-2\int -\frac{1}{2} e^{\boxed{-\frac{1}{2}x}} \, dx$ |
| | $-2e^{-\frac{1}{2}x}$ |

$\square = -\frac{1}{2}x$

$\square' = -\frac{1}{2}$

$\int_0^1 x^2 e^{\boxed{-\frac{1}{2}x}} \, dx$

$\square = -\frac{1}{2}x$

$\square' = -\frac{1}{2}$

reject.

$$u\int v\,dx - \int\left[\frac{du}{dx} \times \int v\,dx\right] dx$$

$$(x^2)(-2e^{-\frac{1}{2}x}) - \int\left[2x \times -2e^{-\frac{1}{2}x}\right] dx$$

$$-2x^2 e^{-\frac{1}{2}x} + 4\int \underbrace{x}_{u} \underbrace{e^{-\frac{1}{2}x}}_{v} \, dx$$

| DIFF OF U | INTEG OF V |
|---|---|
| $x \longrightarrow 1$ | $\int e^{-\frac{1}{2}x} dx = -2e^{-\frac{1}{2}x}$ |

$$u\int v\,dx - \int\left[\frac{du}{dx} \times \int v\,dx\right] dx$$

$$(x)(-2e^{-\frac{1}{2}x}) - \int\left[1 \times -2e^{-\frac{1}{2}x}\right] dx$$

$$-2xe^{-\frac{1}{2}x} + 2\int e^{-\frac{1}{2}x} dx$$

$$-2xe^{-\frac{1}{2}x} + 2\left(-2e^{-\frac{1}{2}x}\right)$$

$$-2x^2 e^{-\frac{1}{2}x} + 4\left(-2x e^{-\frac{1}{2}x} - 4e^{-\frac{1}{2}x}\right)$$

$$-2x^2 e^{-\frac{1}{2}x} - 8x e^{-\frac{1}{2}x} - 16 e^{-\frac{1}{2}x}$$

$$\left| -2e^{-\frac{1}{2}x}\left(x^2+4x+8\right) \right|_0^1$$

$$\left| \left(-2e^{-\frac{1}{2}(1)}\left(1^2+4(1)+8\right)\right) - \left(-2e^{-\frac{1}{2}(0)}\left(0^2+4(0)+8\right)\right) \right|$$

$$-2e^{-\frac{1}{2}}(13)+2(1)(8)$$

$$\boxed{-26e^{-\frac{1}{2}}+16}$$

1

$\boxed{A}$ $y=0$

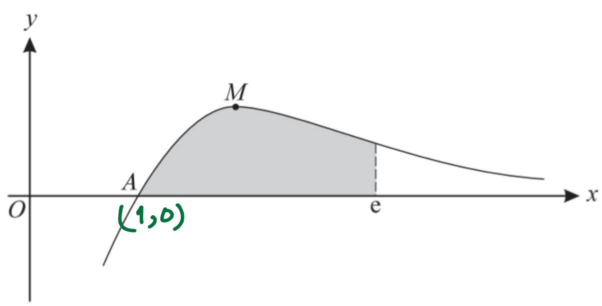
$0=\dfrac{\ln x}{x^2}$

$0=\ln x$

$e$

$x=e^0$

$x=1$




The diagram shows the curve $y=\dfrac{\ln x}{x^2}$ and its maximum point $M$. The curve cuts the $x$-axis at $A$.

**(i)** Write down the $x$-coordinate of $A$. $=1$ [1]

**(ii)** Find the exact coordinates of $M$. [5]

**(iii)** Use integration by parts to find the exact area of the shaded region enclosed by the curve, the $x$-axis and the line $x=e$. [5]

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$$y = \frac{\ln x}{x^2}$$

$$\frac{dy}{dx} = \frac{x^2\left(\frac{1}{x}\times 1\right) - \ln x\,(2x)}{(x^2)^2}$$

$$0 = \frac{x - 2x\ln x}{x^4}$$

$$0 = x(1 - 2\ln x)$$

we know $x$ is non zero hence we can divide.

$$0 = 1 - 2\ln x$$

$$2\ln x = 1$$

$$\ln x = \frac{1}{2}$$

$$x = e^{\frac{1}{2}} \qquad , \qquad y = \frac{\ln e^{\frac{1}{2}}}{(e^{\frac{1}{2}})^2} = \frac{\frac{1}{2}\ln e}{e}$$

$$x = \sqrt{e} \qquad\qquad = \frac{1}{2e}$$

$$M = \left(\sqrt{e}, \frac{1}{2e}\right)$$

(iii) $$\int_1^e \underbrace{x^{-2}}_{v}\,\underbrace{\ln x}_{u}\,dx$$

| DIFF OF $u$ | INTEG OF V |
| --- | --- |
| $\ln x \longrightarrow \frac{1}{x}\times 1 = \frac{1}{x}$ | $\int x^{-2}dx = \frac{x^{-1}}{-1} = \boxed{-\frac{1}{x}}$ |

$$u\int v\,dx - \int\left[\frac{du}{dx}\times\int v\,dx\right]dx$$

$$(\ln x)\left(-\frac{1}{x}\right) - \int\left[\frac{1}{x}\times\frac{-1}{x}\right]dx$$

$$-\frac{1}{x}\ln x + \int x^{-2}\,dx$$

$$-\frac{1}{x}\ln x + \left(-\frac{1}{x}\right)$$

$$\left|\frac{-1}{x}\left(\ln x+1\right)\right|_{1}^{e}$$

$$\left|\left(-\frac{1}{e}\left(\ln e+1\right)\right) - \left(\frac{-1}{1}\left(\ln 1+1\right)\right)\right|$$

$$\frac{-1}{e}(2) + 1(1)$$

$$-\frac{2}{e} + 1$$

$$\boxed{1-\frac{2}{e}}$$

TYPE3: (a) Trig $R\cos(\theta \pm \alpha)$
(b) Integrate.

**1** **(i)** Express $\cos\theta + (\sqrt{3})\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$, giving the exact values of $R$ and $\alpha$. [3]

**(ii)** Hence show that $\int_0^{\frac{1}{2}\pi} \frac{1}{(\cos\theta + (\sqrt{3})\sin\theta)^2}\,d\theta = \frac{1}{\sqrt{3}}$. [4]

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(i) $\cos\theta + \sqrt{3}\sin\theta \equiv R\cos(\theta - \alpha)$

$R[\cos\theta\cos\alpha + \sin\theta\sin\alpha]$

$\cos\theta + \sqrt{3}\sin\theta \equiv R\cos\alpha\cos\theta + R\sin\alpha\sin\theta$

$R\sin\alpha = \sqrt{3}$ , $R\cos\alpha = 1$

$\frac{R\sin\alpha}{R\cos\alpha} = \frac{\sqrt{3}}{1}$

$\tan\alpha = \sqrt{3}$

$\boxed{\alpha = \frac{\pi}{3}}$

$R^2\sin^2\alpha = (\sqrt{3})^2$

$+ \; R^2\cos^2\alpha = 1^2$

$R^2\sin^2\alpha + R^2\cos^2\alpha = 3+1$

$R^2(\sin^2\alpha + \cos^2\alpha) = 4$

$R^2(1) = 4$

$R = 2$.

$\cos\theta + \sqrt{3}\sin\theta \longrightarrow 2\cos\left(\theta - \frac{\pi}{3}\right)$

(ii) $\int_0^{\frac{1}{2}\pi} \frac{1}{(\cos\theta + (\sqrt{3})\sin\theta)^2}\,d\theta = \frac{1}{\sqrt{3}}$.

$\int \frac{1}{\left(2\cos\left(\theta - \frac{\pi}{3}\right)\right)^2}\,d\theta$

$$\int \frac{1}{4\cos^2\left(\theta - \frac{\pi}{3}\right)}\, d\theta$$

$$\frac{1}{4}\int \frac{1}{\cos^2\left(\theta - \frac{\pi}{3}\right)}\, d\theta$$

Power of cos wont be operator.

$$\frac{1}{4}\int \left[\boxed{\cos\left(\theta - \frac{\pi}{3}\right)}\right]^{-2} dx$$

$\square = \cos\left(\theta - \frac{\pi}{3}\right)$

$\square' = -\sin\left(\theta - \frac{\pi}{3}\right)$

reject.

$$\frac{1}{4}\int 1 \sec^2\left(\boxed{\theta - \frac{\pi}{3}}\right) d\theta$$

$\square = \theta - \frac{\pi}{3}$

$\square' = 1$

$$\left|\frac{1}{4}\tan\left(\theta - \frac{\pi}{3}\right)\right|_{0}^{\frac{\pi}{2}}$$

$$\left|\frac{1}{4}\tan\left(\frac{\pi}{2} - \frac{\pi}{3}\right) - \frac{1}{4}\tan\left(0 - \frac{\pi}{3}\right)\right|$$

$$\frac{1}{4}\tan\left(\frac{\pi}{6}\right) - \frac{1}{4}\tan\left(\frac{-\pi}{3}\right)$$

$\tan(-\theta) = -\tan\theta$

$$\frac{1}{4}\left(\frac{1}{\sqrt{3}}\right) - \frac{1}{4}\left(-\tan\left(\frac{\pi}{3}\right)\right)$$

$$\frac{1}{4\sqrt{3}} + \frac{1\sqrt{3}^{\times\sqrt{3}}}{4_{\times\sqrt{3}}}$$

$$\frac{1+3}{4\sqrt{3}} = \frac{4}{4\sqrt{3}} = \frac{1}{\sqrt{3}}$$ shown.

TYPE 4: (a) Trig identity proving
(b) Integrate.
(Always use substitution from the identity above).

**5** **(i)** Prove that $\cot\theta + \tan\theta \equiv 2\operatorname{cosec} 2\theta$. $\longrightarrow$ $\operatorname{cosec} 2\theta = \dfrac{\cot\theta + \tan\theta}{2}$ [3]

**(ii)** Hence show that $\displaystyle\int_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} \operatorname{cosec} 2\theta \, d\theta = \tfrac{1}{2}\ln 3$. [4]

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(ii) $$\int \operatorname{cosec} 2\theta \, d\theta$$

$$\int \frac{\cot\theta + \tan\theta}{2} \, d\theta$$

$$\frac{1}{2}\left[\int \cot\theta \, d\theta + \int \tan\theta \, d\theta\right]$$

$$\frac{1}{2}\left[\int \frac{\cos\theta}{\boxed{\sin\theta}} \, d\theta + (-1)\int \frac{-\sin\theta}{\boxed{\cos\theta}} \, d\theta\right]$$

$\square = \sin\theta$ $\qquad$ $\square = \cos\theta$

$\square' = \cos\theta$ $\qquad$ $\square' = -\sin\theta$

$$\frac{1}{2}\left[\ln|\sin\theta| - \ln|\cos\theta|\right]$$

$$\frac{1}{2}\ln\left|\frac{\sin\theta}{\cos\theta}\right|$$

$$\left|\frac{1}{2}\ln|\tan\theta|\right|_{\frac{\pi}{6}}^{\frac{\pi}{3}}$$

$$\frac{1}{2}\ln\left[\tan\frac{\pi}{3}\right] - \frac{1}{2}\ln\left[\tan\frac{\pi}{6}\right]$$

$$\frac{1}{2}\ln(\sqrt{3}) - \frac{1}{2}\ln\left(\frac{1}{\sqrt{3}}\right)$$

$$\frac{1}{2}\ln 3^{\frac{1}{2}} - \frac{1}{2}\ln 3^{-\frac{1}{2}}$$

$$\frac{1}{4}\ln 3 - \frac{1}{2}\left(\frac{-1}{2}\right)\ln 3$$

$$\frac{1}{4}\ln 3 + \frac{1}{4}\ln 3 = \boxed{\frac{1}{2}\ln 3}$$

3 **(i)** Prove the identity $\cos 4\theta + 4\cos 2\theta \equiv 8\cos^4\theta - 3$. [4]

**(ii)** Hence

**(a)** solve the equation $\cos 4\theta + 4\cos 2\theta = 1$ for $-\frac{1}{2}\pi \leqslant \theta \leqslant \frac{1}{2}\pi$, [3]

**(b)** find the exact value of $\int_0^{\frac{1}{4}\pi} \cos^4\theta \, d\theta$. [3]

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(ii) (b) Substitution from identity

$$\cos 4\theta + 4\cos 2\theta = 8\cos^4\theta - 3$$

$$\cos^4\theta = \frac{\cos 4\theta + 4\cos 2\theta + 3}{8}$$

$$\int \cos^4\theta \, d\theta$$

$$\int\left(\frac{\cos 4\theta + 4\cos 2\theta + 3}{8}\right) d\theta$$

$$\frac{1}{8}\left[\frac{1}{4}\int 4\,\cos\boxed{4\theta}\,d\theta + \frac{4}{2}\int 2\,\cos\boxed{2\theta}\,d\theta + \int 3\,d\theta\right]$$

$$\frac{1}{8}\left[\frac{1}{4}\sin 4\theta + 2\sin 2\theta + 3\theta\right]_0^{\frac{\pi}{4}}$$

$$\left|\frac{1}{8}\left(\frac{1}{4}\sin\left(\cancel{4}\times\frac{\pi}{\cancel{4}}\right) + 2\sin\left(\cancel{2}\times\frac{\pi}{2\cancel{4}}\right) + 3\left(\frac{\pi}{4}\right)\right) - \frac{1}{8}\left(\frac{1}{4}\sin 4(0) + 2\sin(2)(0) + 3(0)\right)\right|$$

$$\frac{1}{8}\left(\frac{1}{4}(0) + 2(1) + \frac{3\pi}{4}\right) - \frac{1}{8}\left(0 + 0 + 0\right)$$

$$\frac{1}{8}\left(2 + \frac{3\pi}{4}\right) = \frac{1}{8}\left(\frac{8 + 3\pi}{4}\right)$$

$$= \boxed{\frac{8 + 3\pi}{32}}$$

# TYPES: Single part Questions.
(usually at start of each P3)

3 Show that $\int_0^{\pi} \underbrace{x^2}_{u} \underbrace{\sin x}_{v} \, dx = \pi^2 - 4.$ [5]

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| DIFF OF $u$ | INTEG OF V |
|---|---|
| $x^2 \longrightarrow 2x$ | $\int 1 \sin \boxed{x} \, dx = \boxed{-\cos x}$ |

$$u \int v \, dx - \int \left[ \frac{du}{dx} \times \int v \, dx \right] dx$$

$$(x^2)(-\cos x) - \int \left[ 2x \times (-\cos x) \right] dx$$

$$-x^2 \cos x + 2 \int \underbrace{x}_{u} \underbrace{\cos x}_{v} \, dx$$

| Diff of $u$ | INTEG OF V |
|---|---|
| $x \longrightarrow 1$ | $\int 1 \cos \boxed{x} \, dx = \sin x$ |

$$u \int v \, dx - \int \left[ \frac{du}{dx} \times \int v \, dx \right] dx$$

$$(x)(\sin x) - \int \left[ 1 \times \sin x \right] dx$$

$$x \sin x - \int \sin x \, dx$$

$$x \sin x - (-\cos x)$$

$$-x^2 \cos x + 2 (x \sin x + \cos x)$$

$$\Big| -x^2 \cos x + 2x \sin x + 2 \cos x \Big|_0^{\pi}$$

$$\left|\left(-\pi^2\cos\pi + 2\pi\sin\pi + 2\cos\pi\right) - \left(-0^2\cos 0 + 2(0)\sin 0 + 2\cos(0)\right)\right|$$

$$-\pi^2(-1) + 2\pi(0) + 2(-1) - \left(0 + 0 + 2(1)\right)$$

$$\pi^2 \quad -2 \quad -2$$

$$\boxed{\pi^2 - 4}$$

4th Jan Monday

$(4-5) = M1$

$(5-6) = S1$

---

Remaking P3 go in morning 12/1 pm.

## TYPE 6: INTEGRATION WITH SUBSTITUTION:-

(Question will always tell you when to use Substitution).

**Q: BASIC QUESTION TO LEARN PROCESS:**

$$\int_1^2 x\left(2x^2+3\right)^4 dx$$

$u = 2x^2+3$

STEP 1: Diff the substit.

$$\frac{du}{dx} = 4x$$

$$\boxed{dx = \frac{du}{4x}}$$

STEP 2: Start substituting.

$$\int \cancel{x}\,(u)^4 \frac{du}{4\cancel{x}}$$

$$\int \frac{u^4}{4}\, du$$

$$\frac{1}{4}\int u^4\, du$$

$$\frac{1}{4} \times \frac{u^5}{5}$$

$$\left|\frac{u^5}{20}\right|_5^{11}$$

Step 3

| CHANGE OF LIMITS | |
|---|---|
| LOWER | UPPER |
| $u = 2x^2+3$ | $u = 2x^2+3$ |
| $x = 1$ | $x = 2$ |
| $u = 2(1)^2+3$ | $u = 2(2)^2+3$ |
| $u = 5$ | $u = 11$ |

$$\left|\frac{11^5}{20} - \frac{5^5}{20}\right| = \boxed{\phantom{0}}$$

4 **(i)** Use the substitution $x = 2\tan\theta$ to show that

$$\int_0^2 \frac{8}{(4+x^2)^2}\,dx = \int_0^{\frac{1}{4}\pi} \cos^2\theta\,d\theta. \quad [4]$$

**(ii)** Hence find the exact value of

$$\int_0^2 \frac{8}{(4+x^2)^2}\,dx. = \int \cos^2\theta\,d\theta \quad [4]$$

odd/even powers of cos.

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(i) $\int_0^2 \frac{8}{(4+x^2)^2}\,dx$

$x = 2\tan\theta$

$\frac{dx}{d\theta} = 2\sec^2\theta \times 1$

$\boxed{dx = 2\sec^2\theta\,d\theta}$

$\int \frac{8}{(4+(2\tan\theta)^2)^2} \cdot 2\sec^2\theta\,d\theta$

$\int \frac{16\sec^2\theta}{(4+4\tan^2\theta)^2}\,d\theta$

$\int \frac{16\sec^2\theta}{(4(1+\tan^2\theta))^2}\,d\theta$

$\int \frac{16\sec^2\theta}{(4\sec^2\theta)^2}\,d\theta$

$\int \frac{\cancel{16}\cancel{\sec^2\theta}}{\cancel{16}(\sec^2\theta)^{\cancel{2}}}\,d\theta$

| CHANGE OF LIMITS | |
|---|---|
| LOWER | UPPER |
| $x = 2\tan\theta$ | $x = 2\tan\theta$ |
| $x = 0$ | $x = 2$ |
| $0 = 2\tan\theta$ | $2 = 2\tan\theta$ |
| $0 = \tan\theta$ | $1 = \tan\theta$ |
| $\theta = 0$ | $\theta = \frac{\pi}{4}$ |

PROVING.

TRI

$$\int \frac{1}{\sec^2\theta}\, d\theta$$

$$\int_0^{\frac{\pi}{4}} \cos^2\theta\, d\theta$$

15 Let $I = \int_0^1 \frac{\sqrt{x}}{2-\sqrt{x}}\, dx$.

(i) Using the substitution $u = 2 - \sqrt{x}$, show that $I = \int_1^2 \frac{2(2-u)^2}{u}\, du$. [4]

(ii) Hence show that $I = 8\ln 2 - 5$. [4]

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make x-subject

$\sqrt{x} = 2-u$

$x = (2-u)^2$

$u = 2 - \sqrt{x}$

$\frac{du}{dx} = 0 - \frac{1}{2}x^{-\frac{1}{2}}(1)$

$\frac{du}{dx} = -\frac{1}{2\sqrt{x}}$

$-dx = 2\sqrt{x}\, du$

$\boxed{dx = -2\sqrt{x}\, du}$

$$\int_0^1 \frac{\sqrt{x}}{2-\sqrt{x}}\, dx$$

$$\int \frac{\sqrt{x}}{u} \cdot (-2\sqrt{x})\, du$$

$$\int \frac{-2x}{u}\, du$$

$$\int_2^1 \frac{-2(2-u)^2}{u}\, du$$

Flip limits.

$$\int_1^2 \frac{2(2-u)^2}{u}\, dx$$

| CHANGE OF LIMITS | |
|---|---|
| LOWER | UPPER |
| $u = 2-\sqrt{x}$<br>$x = 0$ | $u = 2-\sqrt{x}$<br>$x = 1$ |
| $u = 2-\sqrt{0}$ | $u = 2-\sqrt{1}$ |
| $u = 2$ | $u = 1$ |
| | |

RULE:

$$\int_a^b f(x)\,dx = \int_b^a -f(x)\,dx$$

IF WE FLIP LIMITS, INNER FUNCTION IS MULTIPLIED BY -1.